M.Sc. 2nd Semester (2020)

MEASURE THEORY

(4) <u>Condensation Point</u> :- A point 'x' is said to be a condensation point of a set A, if every neighbourhood containing x contains an infinite number of points of A.

(5) <u>Closed Sets</u> :- A set A is said to be closed if every limiting point of A belongs to the set A itself.

Symbolically, a set A is said to be closed if $D(A) \subset A$.

<u>Examples</u>: (i) $\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots\} = A$, is a closed set, for $D(A) = \{0\} \subset A$.

(ii) A closed interval is a closed set.
For $D([a,b]) = [a,b] \subset [a,b]$.

(iii) Every real number is a limit point so that rational limit point belongs to the set Q.
This $\Rightarrow D(R) = R \subset R$ $D(Q) = R \not\subset Q$
So that R is closed set whereas Q is not.

(6.) Cardially Equivalent :-

A set A is said to be Cardially equivalent to a set B if $\exists$ (at least one) one-one map from A onto B. This fact is donated by the symbol $A \sim B$.

This can also be expressed by saying that:

(i.) A is numerically equivalent to B.

(ii.) A is equivalent to B.

(iii.) A is equipollent to B.

(iv.) A and B have the same power.

Examples :→ (a.) Any two singleton sets are equal.

(b.) Let $N = \{1, 2, 3 \ldots\}$, $E = \{2, 4, 6 \ldots\}$, define a map $f : N \to E$ by the formula $f(n) = 2n$.

Evidently if f is one-one and onto.

Hence $N \sim E$, This example shows that the set N is equivalent to a proper subject of itself. This is a striking property of an infinite set.